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$$\cos\varphi\sin(C+\omega)=\sin\omega[-(x/a)\cos A+(y/b)\cos(\omega-B)]$$

$$\sin\varphi\sin(C+\omega)=\sin\omega[(x/a)\sin A-(y/b)\sin(\omega-B)].$$

Squaring and adding, and transposing the members of the equation, we obtain the required equation of the locus

$$\frac{x^2}{a^2} + \frac{2xy}{ab}\cos(C+\omega) + \frac{y^2}{b^2} = \frac{\sin^2(C+\omega)}{\sin^2\omega}.$$

The locus is therefore an ellipse with center at the intersection of the two lines.

Solved similarly by G. W. GREENWOOD, and G. B. M. ZERR.

CALCULUS.

163. Proposed by F. P. MATZ, So. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Can there be a plane curve the length of which varies *directly as the abscissa and inversely as the ordinate* of any point on the curve?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$$S=mx/y, \quad dS=\frac{mydx-mxdy}{y^2}=\sqrt{(dx^2+dy^2)}.$$

$$\therefore \sqrt{[1+(dx/dy)^2]}=[my(dx/dy)-mx]/y^2.$$

Let $x=vy$, $dx/dy=v+y(dv/dy)=v+yp$, suppose.

$$\therefore \sqrt{[1+(v+yp)^2]}=mp, \text{ or } v+yp=\sqrt{(m^2p^2-1)} \dots (1).$$

$$\text{From (1), } p=\frac{vy \pm \sqrt{(v^2m^2+m^2-y^2)}}{m^2-y^2} = \frac{xy \pm \sqrt{[m^2(x^2+y^2)-y^4]}}{y(m^2-y^2)}.$$

Differentiating (1) with respect to y ,

$$dv/dy=p=p-y(dp/dy)+\frac{m^2p(dp/dy)}{\sqrt{(m^2p^2-1)}}.$$

$$\therefore 2p=\left(\frac{m^2p}{\sqrt{(m^2p^2-1)}}-y\right)\frac{dp}{dy}.$$

$$\therefore \frac{dy}{dp}=\frac{m^2}{2\sqrt{(m^2p^2-1)}}-\frac{y}{2p} \text{ or } \frac{dy}{dp}+\frac{y}{2p}=\frac{m^2}{2\sqrt{(m^2p^2-1)}}.$$

$$\therefore y\sqrt{p}=C+\frac{m^2}{2}\int\frac{\sqrt{p}dp}{\sqrt{(m^2p^2-1)}}=C+\frac{m^2}{2}f(p) \dots (2).$$

The value of p in (2) from (1) gives the primitive, which is the curve desired. By series we get

$$\frac{m^2}{2} \int \frac{\sqrt{p} dp}{\sqrt{(m^2 p^2 - 1)}} = \frac{1}{2\sqrt{p}} \sqrt{(m^2 p^2 - 1)} + \frac{1}{2} m \sqrt{p} \left(1 + \frac{1}{2.3 m^2 p^2} + \frac{1}{2.4.7 m^4 p^4} \right. \\ \left. + \frac{3}{2.4.6.11 m^6 p^6} + \frac{3.5}{2.4.6.8.15 m^8 p^8} + \dots \right).$$

164. Proposed by G. B. M. ZERE, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $m^2 + n^2 = 1$, $m^2 \cos^2 \theta + n^2 \cos^2 \varphi = A$, $a^2 b^2 \sin^2 \theta (m^2 + n^2 \cos^2 \varphi) + a^2 c^2 \cos^2 \theta \cos^2 \varphi$ $+ b^2 c^2 \sin^2 \varphi (n^2 + m^2 \cos^2 \theta) = B$, $\sqrt{(1 - m^2 \sin^2 \theta)} = \Delta(\theta)$, $\sqrt{(1 - n^2 \sin^2 \varphi)} = \Delta(\varphi)$

prove that $\int_0^{\frac{1}{2}\pi} \int_0^{\frac{1}{2}\pi} \frac{A B d\theta d\varphi}{\Delta(\theta) \Delta(\varphi)} = \left(\frac{1}{8}\pi\right) (a^2 b^2 + a^2 c^2 + b^2 c^2)$.

Solution by the PROPOSER.

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1.$$

$$\int ds = \iint \sqrt{[1 + (dz/dx)^2 + (dz/dy)^2]} dx dy = c^2 \iint \frac{\sqrt{[z^2/c^4 + x^2/a^4 + y^2/b^4]} dx dy}{z}.$$

Let p = perpendicular from center on tangent plane.

$$\therefore 1/p = \sqrt{[z^2/c^4 + x^2/a^4 + y^2/b^4]}.$$

$$\text{Let } (b/a)\sqrt{[a^2 - x^2]} = y', \quad (a/b)\sqrt{[b^2 - y^2]} = x'.$$

$$D = \int \frac{ds}{p} = c^2 \iint \frac{[z^2/c^4 + x^2/a^4 + y^2/b^4] dx dy}{z} = \frac{1}{c^2} \int_0^a \int_0^{y'} \frac{z dx dy}{z} + \frac{c^2}{a^4} \int_0^a \int_0^{y'} \frac{y^2 x^2}{z} dx dy \\ + \frac{c^2}{b^4} \int_0^b \int_0^{x'} \frac{(y^2/z) dy dx}{z} = \frac{1}{abc} \int_0^a \int_0^{y'} \sqrt{[a^2 b^2 - b^2 x^2 - a^2 y^2]} dx dy \\ + \frac{abc}{a^4} \int_0^a \int_0^{y'} \frac{x^2 dx dy}{\sqrt{[a^2 b^2 - b^2 x^2 - a^2 y^2]}} + \frac{abc}{b^4} \int_0^b \int_0^{x'} \frac{y^2 dy dx}{\sqrt{[a^2 b^2 - b^2 x^2 - a^2 y^2]}} \\ = \frac{1}{8}\pi (ab/c + bc/a + ac/b) = \frac{\pi}{6abc} (a^2 b^2 + b^2 c^2 + a^2 c^2).$$

$$\text{Let } x = a \sin \varphi_1 \sqrt{(1 - m^2 \sin^2 \theta)}, \quad y = b \cos \theta \cos \varphi, \quad z = c \sin \theta \sqrt{(1 - n^2 \sin^2 \varphi)},$$

$$m^2 + n^2 = 1; \quad dx/d\varphi = a \cos \varphi_1 \sqrt{(1 - m^2 \sin^2 \theta)}, \quad dx/d\theta = -\frac{am^2 \sin \varphi \sin \theta \cos \theta}{\sqrt{(1 - m^2 \sin^2 \theta)}}, \quad dy/d\varphi = \\ -b \cos \theta \sin \varphi, \quad dy/d\theta = -b \sin \theta \cos \varphi; \quad dx dy = (dx/d\theta)(dy/d\varphi) - (dx/d\varphi)(dy/d\theta)$$